Hall Ticket Number:

Code No. : 13154 S (D) N

VASAVI COLLEGE OF ENGINEERING *(AUTONOMOUS)*, HYDERABAD

Accredited by NAAC with A++ Grade

B.E. III-Semester Supplementary Examinations, August-2023

Linear Algebra (OE-I)

Time: **3 hours** Max. Marks: **60**

Note: Answerallquestions fromPart-A and any FIVE from Part-B

Part-A (10× 2 = 20 Marks)

Q. No.	Stem of the question	M	L	CO	PO
1.	Let V=ℝ. Define addition and scalar multiplication by **a**⊕**b**=2a+2b and kʘ**a**=ka. Show that V is not a vector space.	2	1	1	1, 2, 12
2.	Define subspace of a vector space. Give example.	2	1	1	1, 2, 12
3.	Show that $T: P_3 \to P_2$ defined by $T(p(x)) = p'(x)$ is a linear transformation.	2	2	2	1, 2, 12
4.	If $T: V \to W$ is a linear transformation define Coordinates of $\boldsymbol{v} \in V$	2	1	2	1, 2, 12
5.	Define range and null space of a linear transformation.	2	1	3	1, 2, 12
6.	State Rank-Nullity theorem.	2	1	3	1, 2, 12
7.	Determine whether $V = \mathbb{R}^2$ for $\boldsymbol{u}, \boldsymbol{v} \in V$ defined by $\langle \boldsymbol{u}, \boldsymbol{v} \rangle = u_1v_1$-$2u_1v_2 - 2u_2v_1 + 3u_2v_2$ is inner product space.	2	2	4	1, 2, 12
8.	Describe the set of all vectors in $\mathbb{R}^2$ that are orthogonal to $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$	2	2	4	1, 2, 12
9.	Let $S = \{1, x-3, x^2+2x, 2x^2+3x+5\}$.Find whether S is linearly independent.	2	2	1	1, 2, 12
10.	Define Linear Transformation between two Vector Spaces.	2	1	2	1, 2, 12
	Part-B (5×8 = 40 Marks)				
11. a)	Determine if the matrix $\begin{bmatrix} -2 & 1 \\ 6 & 5 \end{bmatrix}$ is in the span of $S = \left\{ \begin{bmatrix} 1 & 1 \\ 0 & -1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 2 & 1 \end{bmatrix}, \begin{bmatrix} 1 & -1 \\ -4 & -3 \end{bmatrix} \right\}$	4	2	1	1, 2, 12
b)	Find a basis for the span(S) as a subspace of $\mathbb{R}^3$, $S = \left\{ \begin{bmatrix} -2 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -3 \end{bmatrix}, \begin{bmatrix} -3 \\ -3 \\ -2 \end{bmatrix}, \begin{bmatrix} 1 \\ 2 \\ -2 \end{bmatrix} \right\}$	4	2	1	1, 2, 12
12. a)	Let $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(\boldsymbol{u}) = A\boldsymbol{u}$ where $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 3 \\ 1 & 3 & 2 \end{bmatrix}$ find $T(3\boldsymbol{e_1} - 4\boldsymbol{e_2} + 6\boldsymbol{e_3})$	4	2	2	1, 2, 12
b)	Define $T: \mathbb{R}^3 \to \mathbb{R}^2$ by $T\left(\begin{bmatrix} x \\ y \\ z \end{bmatrix}\right) = \begin{bmatrix} x+2y+z \\ -x+5y+z \end{bmatrix}$. Find all vectors that are mapped to **0**.	4	4	2	1, 2, 12

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13. a)	Find a basis for the null space of the linear transformation $T:\mathbb{R}^3 \to \mathbb{R}^3$ defined by $T\left(\begin{bmatrix}x\\y\\z\end{bmatrix}\right) = \begin{bmatrix}x+2z\\2x+y+3z\\x-y+3z\end{bmatrix}$	4	2	3	1, 2, 12
b)	If a linear transformation $T: P_2 \to P_2$ is defined by $T(p(x)) = p'(x) + p(x)$, find the matrix representation of T relative to the ordered bases B and B'. Given $B = \{1-x-x^2, 1, 1+x^2\}$ and $B' = \{-1+x, -1+x+x^2, x\}$	4	4	3	1, 2, 12
14. a)	The inner product on P_2 defined by $\langle p, q\rangle = \int_0^1 p(x)q(x)dx$. If $(x) = x^2 - x + 1$, $q(x) = 3x - 1$ find $p - Proj_q p$ and verify that it is orthogonal to q.	4	3	4	1, 2, 12
b)	Find the orthogonal complement of W in $\mathbb{R}^n$ with the standard inner product $W = \textbf{span}\left\{\begin{bmatrix}-3\\1\\-1\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right\}$	4	2	4	1, 2, 12
15. a)	Find the transition matrix between the ordered basis B_1 and B_2 ; then given $[v]_{B_1}$ find $[v]_{B_2}$ Where $B_1 = \left\{\begin{bmatrix}-2\\1\\0\end{bmatrix}, \begin{bmatrix}1\\0\\2\end{bmatrix}, \begin{bmatrix}-1\\1\\0\end{bmatrix}\right\}$ and $B_2 = \left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\1\\2\end{bmatrix}, \begin{bmatrix}-1\\-1\\0\end{bmatrix}\right\}$ $v]_{B_1} = \begin{bmatrix}2\\1\\1\end{bmatrix}$	4	4	1	1, 2, 12
b)	Define $T: P_3 \to \mathbb{R}^2$ by $T(ax^3 + bx^2 + cx + d) = \begin{bmatrix}-a-b+1\\c+d\end{bmatrix}$. Is T a linear transformation?	4	2	2	1, 2, 12
16. a)	Let $T: P_2 \to P_2$ defined by $T(ax^2 + bx + c) = cx^2 + bx - b$, find basis of R(T).	4	4	3	1, 2, 12
b)	Let $B = \left\{\begin{bmatrix}1\\0\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\1\end{bmatrix}, \begin{bmatrix}0\\-1\\-1\end{bmatrix}\right\}$. Use the standard inner product on $\mathbb{R}^n$ to find an orthonormal basis for $\mathbb{R}^n$ applying Gram-Schmidt process.	4	3	4	1, 2, 12
17.	Answer any ***two*** of the following:				
a)	Find a basis for the vector space V that contains the vectors $\left\{\begin{bmatrix}2\\-1\\3\end{bmatrix}, \begin{bmatrix}1\\0\\2\end{bmatrix}\right\}, V = \mathbb{R}^3$	4	3	1	1, 2, 12
b)	State and prove that sum of two linear transformations is a linear transformation.	4	1	2	1, 2, 12
c)	Find a basis for the range of the linear transformation $T: \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(v) = \begin{bmatrix}1 & 1 & 2\\0 & 1 & -1\\2 & 0 & 1\end{bmatrix} v$	4	3	3	1, 2, 12

M : Marks; L: Bloom's Taxonomy Level; CO; Course Outcome; PO: Programme Outcome

i)	Blooms Taxonomy Level – 1	20%
ii)	Blooms Taxonomy Level – 2	40%
iii)	Blooms Taxonomy Level – 3 & 4	40%

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